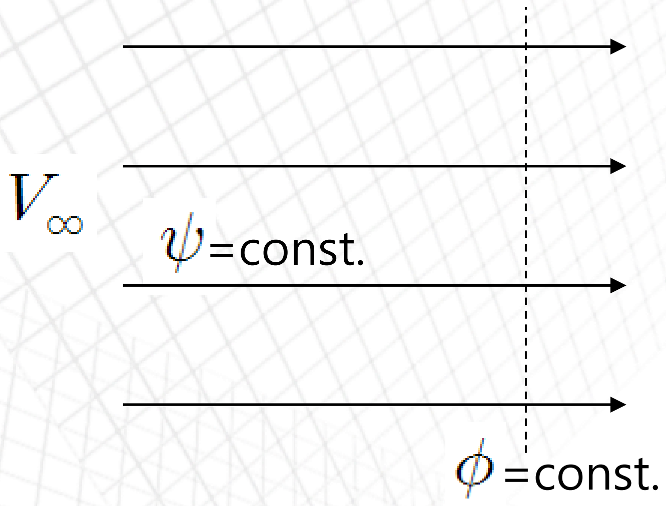
< 3.9. Uniform Flow >



$$u = V_\infty = \frac{\partial \phi}{\partial x} \quad \rightarrow \quad \phi = V_\infty x + f(y)$$

$$v = 0 = \frac{\partial \phi}{\partial y} \quad \rightarrow \quad \phi = f(x) + const$$

$$\phi = V_\infty x + const$$

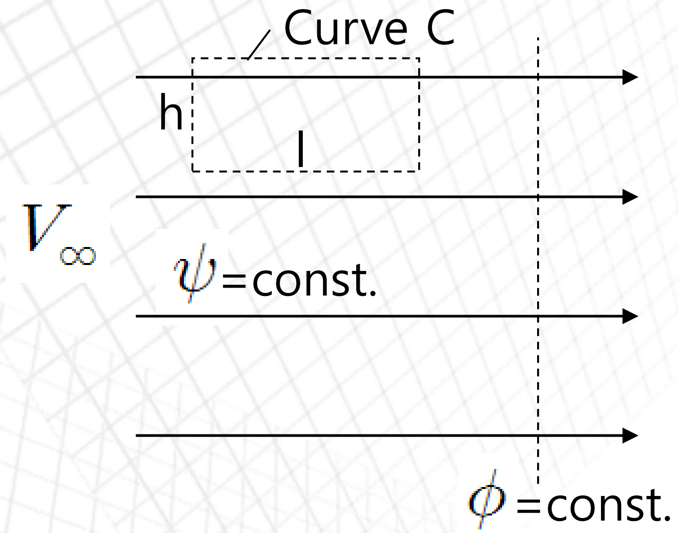
can be set to zero

$$\therefore \phi = V_\infty x$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = V_\infty \quad \rightarrow \quad \psi = V_\infty y + const$$

zero

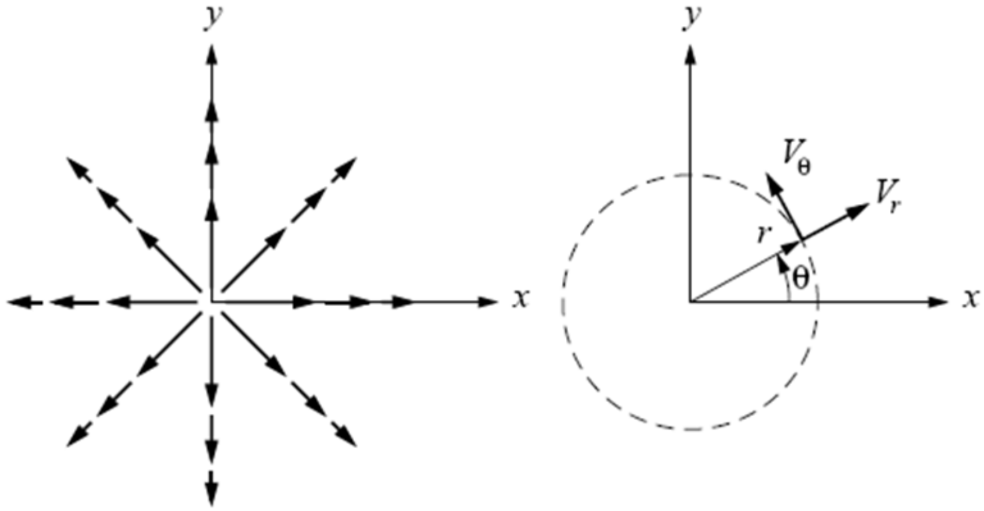
< 3.9. Uniform Flow >



Evaluate Γ in a uniform flow

$$\Gamma = \oint \vec{V} \cdot \vec{ds} = 0 \cdot h + V_\infty l - 0 \cdot h + (-V_\infty) l = 0$$

< 3.10. Source/Sink Flow >



Mass flow

$$\dot{m} = 2\pi r l \rho V_r$$

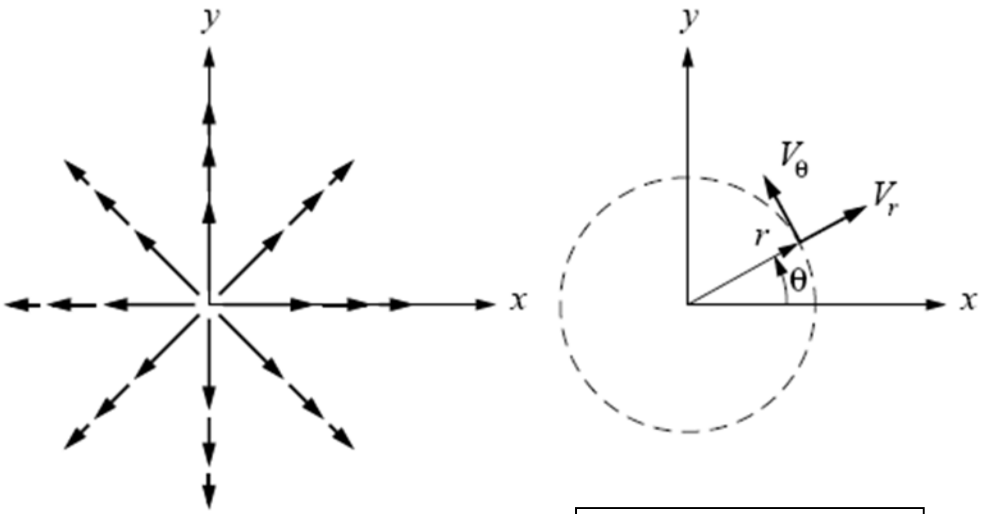
Volume flow rate

$$\dot{v} = \dot{m}/\rho = 2\pi r l V_r$$

$$\longrightarrow \quad V_r = \frac{\frac{\dot{v}}{l}}{2\pi r} = \frac{\Lambda}{2\pi r}$$

$\Lambda = \dfrac{\dot{v}}{l}$: Source strength
Volume flow rate per unit length

< 3.10. Source/Sink Flow >



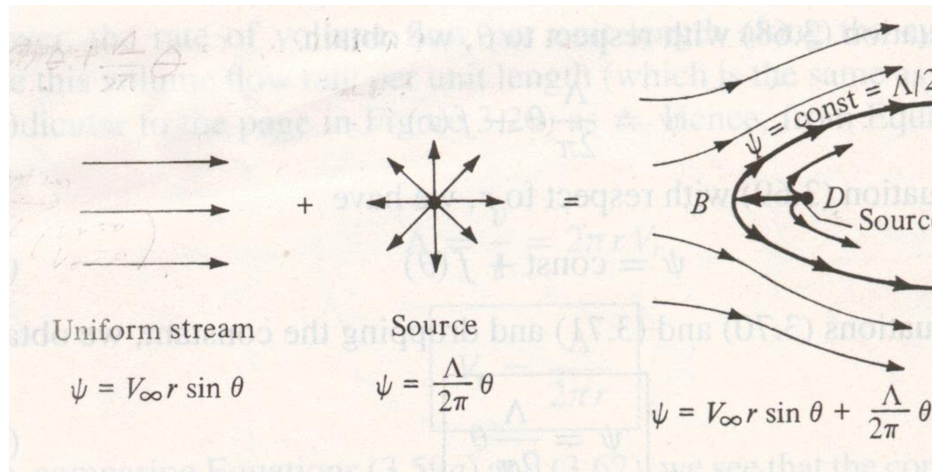
$$\frac{\partial \phi}{\partial r} = V_r = \frac{\Lambda}{2\pi r} \rightarrow \boxed{\phi = \frac{\Lambda}{2\pi} \ln r}$$

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\Lambda}{2\pi r} \rightarrow \boxed{\psi = \frac{\Lambda}{2\pi}\theta}$$

For sink flow, set Λ to $-\Lambda$

< 3.11. A Uniform Flow with a Source and Sink>

❖ Uniform flow + Souce



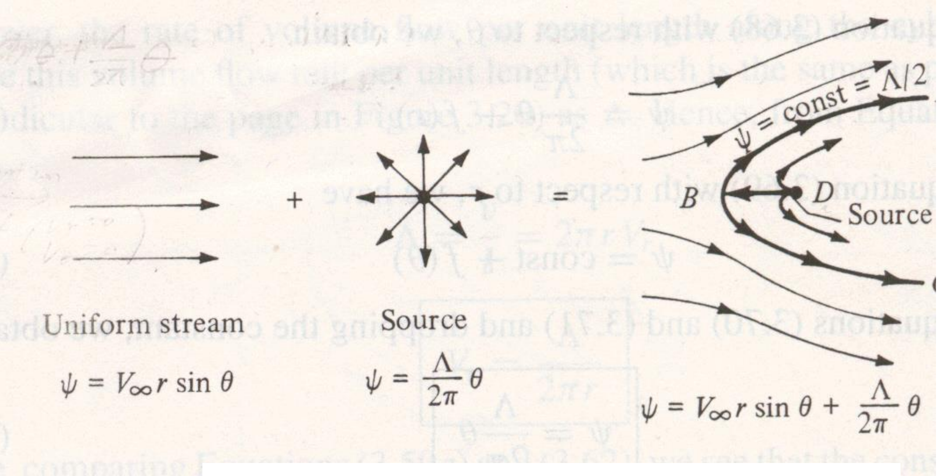
$$\psi = V_\infty r sin\theta + \frac{\Lambda}{2\pi}\theta$$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty \cos\theta + \frac{\Lambda}{2\pi r}$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty \sin\theta$$

< 3.11. A Uniform Flow with a Source and Sink>

❖ Uniform flow + Souce



- At the stagnation point

$$V_\infty \cos\theta + \frac{\Lambda}{2\pi r} = 0$$

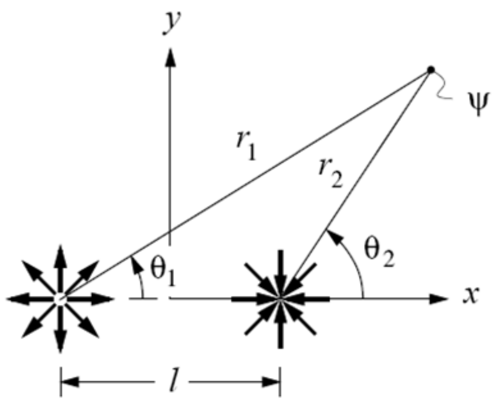
$$V_\infty \sin\theta = 0$$

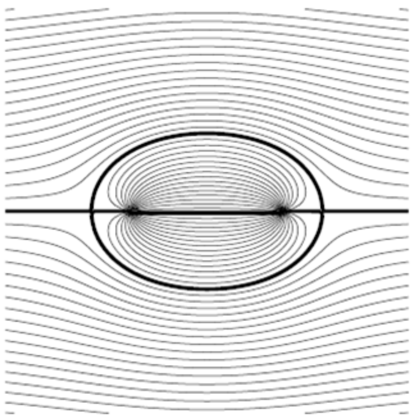
$$\longrightarrow \quad (r,\theta)_{STAG} = \left(\frac{\Lambda}{2\pi V_\infty}, \pi\right)$$

- Streamline going through the stagnation point → $\psi = \frac{\Lambda}{2}$
- So, the body surface can be replaced by a streamline

< 3.11. A Uniform Flow with a Source and Sink>

❖ Uniform flow + Souce + Sink





Rankine oval

$$\psi = V_\infty r sin\theta + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

< 3.12. Doublet Flow >

❖ Uniform flow + Souce + Sink

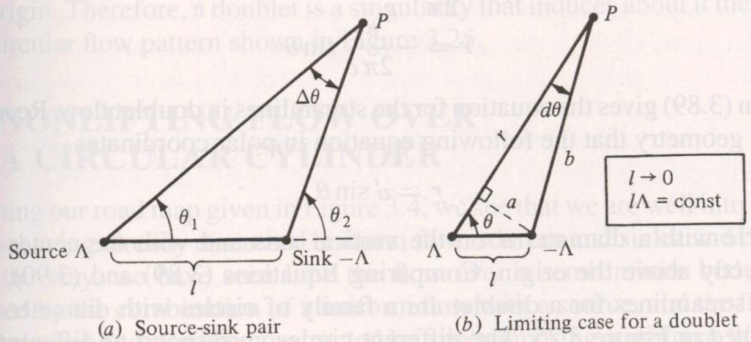
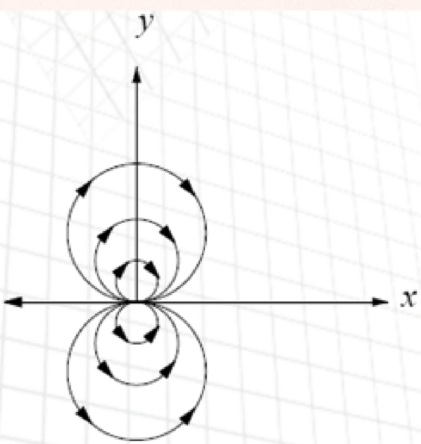


Figure 3.24 How a source-sink pair approaches a doublet in the limiting case.



$$\Delta\theta = \theta_2 - \theta_1$$

$$\psi = \lim_{l\to 0}\left(-\frac{\Lambda}{2\pi}d\theta\right)$$

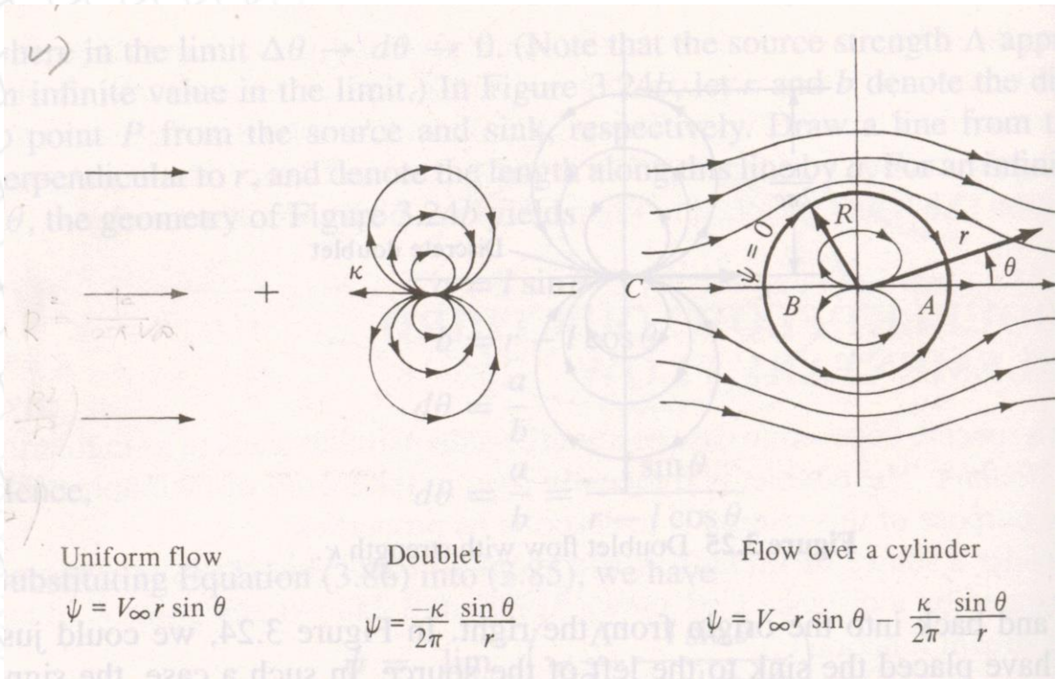
$$d\theta \simeq \frac{l\sin\theta}{r - l\cos\theta}$$

$$\psi = \lim_{l\to 0}\left(-\frac{\Lambda}{2\pi}\frac{l\sin\theta}{(r - l\cos\theta)}\right) = \boxed{-\frac{\mu\sin\theta}{2\pi r}}$$

$$(\mu = \Lambda l : doublet\ \ strength)$$

$$\boxed{\phi = \frac{\mu}{2\pi}\frac{\cos\theta}{r}}$$

< 3.13. Nonlifting Flow over a Circular Cylinder >



$$\psi = V_\infty r sin\theta - \frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$= V_\infty r sin\theta \left(1 - \frac{\mu}{2\pi V_\infty r^2}\right)$$

< 3.13. Nonlifting Flow over a Circular Cylinder >

Let $R^2 = \dfrac{\mu}{2\pi V_\infty} \longrightarrow \psi = V_\infty r sin\theta(1-\dfrac{R^2}{r^2})$

$$\rightarrow V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty \cos\theta(1-\frac{R^2}{r^2})$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = -(1+\frac{R^2}{r^2})V_\infty \sin\theta$$

At the stagnation point, $V_r = V_\theta = 0 \rightarrow (r,\theta) = (R,\theta)$ & (R,π)

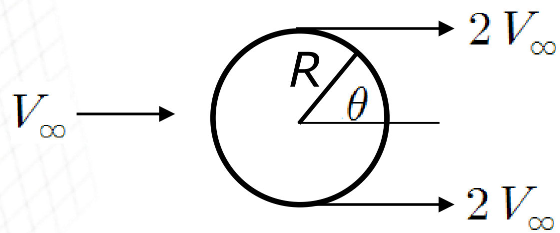
Stagnation streamline : $\psi = 0$

< 3.13. Nonlifting Flow over a Circular Cylinder >

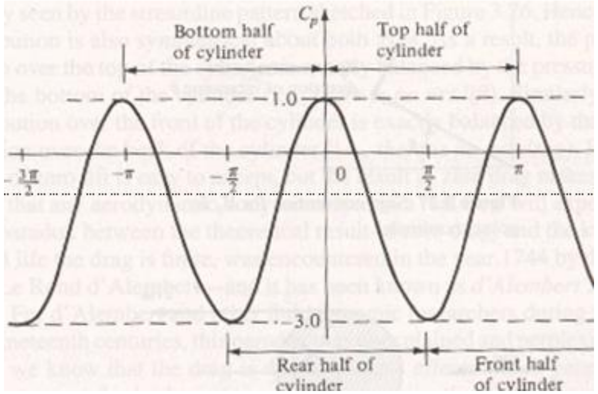
Now, check the pressure distribution at the surface of the cylinder.

➔ $V_r = 0$

$V_\theta = -2V_\infty \sin\theta$



➔ $C_p = 1 - \left(\frac{V^2}{V_\infty^2}\right) = 1 - 4\sin^2\theta$



➔ No Lift
➔ No Drag

* In the real situation, no lift is acceptable.
But no drag makes non-sense. → **d'Alembert Paradox** in 18c
what happens in real life? → the role of **viscosity** makes drag